ADVANCED SUBSIDIARY GCE

MATHEMATICS 4721

Core Mathematics 1

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 4721
- List of Formulae (MF1)

Other materials required:
None

Monday 10 January 2011
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES
These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are **not** permitted to use a calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES
This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The printed answer book consists of **12** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

No calculator can be used for this paper

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1 The points A and B have coordinates (6, 1) and (−2, 7) respectively.

(i) Find the length of AB. **[2]**

(ii) Find the gradient of the line AB. **[2]**

(iii) Determine whether the line $4x - 3y - 10 = 0$ is perpendicular to AB. **[3]**

2 Given that

$$(x - p)(2x^2 + 9x + 10) = (x^2 - 4)(2x + q)$$

for all values of x, find the constants p and q. **[3]**

3 Express each of the following in the form 8^p:

(i) $\sqrt{8}$, **[1]**

(ii) $\frac{1}{64}$, **[1]**

(iii) $2^6 \times 2^2$. **[3]**

4 By using the substitution $u = (3x - 2)^2$, find the roots of the equation

$$(3x - 2)^4 - 5(3x - 2)^2 + 4 = 0.$$ **[6]**

5 **(i)** Sketch the curve $y = -x^3$. **[2]**

(ii) The curve $y = -x^3$ is translated by 3 units in the positive x-direction. Find the equation of the curve after it has been translated. **[2]**

(iii) Describe a transformation that transforms the curve $y = -x^3$ to the curve $y = -5x^3$. **[2]**

6 Given that $y = \frac{5}{x^2} - \frac{1}{4x} + x$, find

(i) $\frac{dy}{dx}$, **[4]**

(ii) $\frac{d^2y}{dx^2}$. **[2]**

7 **(i)** Express $4x^2 + 12x - 3$ in the form $p(x + q)^2 + r$. **[4]**

(ii) Solve the equation $4x^2 + 12x - 3 = 0$, giving your answers in simplified surd form. **[4]**

(iii) The quadratic equation $4x^2 + 12x - k = 0$ has equal roots. Find the value of k. **[3]**

8 **(i)** Find the equation of the tangent to the curve $y = 7 + 6x - x^2$ at the point P where $x = 5$, giving your answer in the form $ax + by + c = 0$. **[6]**

(ii) This tangent meets the x-axis at Q. Find the coordinates of the mid-point of PQ. **[3]**

(iii) Find the equation of the line of symmetry of the curve $y = 7 + 6x - x^2$. **[2]**

(iv) State the set of values of x for which $7 + 6x - x^2$ is an increasing function. **[2]**

9 A circle with centre C has equation $x^2 + y^2 - 8x - 2y - 3 = 0$.

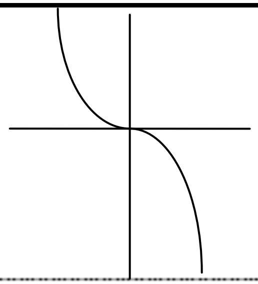
(i) Find the coordinates of C and the radius of the circle. **[3]**

(ii) Find the values of k for which the line $y = k$ is a tangent to the circle, giving your answers in simplified surd form. **[3]**

(iii) The points S and T lie on the circumference of the circle. M is the mid-point of the chord ST. Given that the length of CM is 2, calculate the length of the chord ST. **[3]**

(iv) Find the coordinates of the point where the circle meets the line $x - 2y - 12 = 0$. **[6]**

1 (i)	$\sqrt{(-2-6)^2+(7-1)^2}$ $= 10$	M1 A1	2	Use of $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	3 out of 4 substitutions correct Look out for no square root, $(x_2+x_1)^2$ etc. **M0**
(ii)	$\frac{7-1}{-2-6}$ $=-\frac{3}{4}$	M1 A1	2	uses $\frac{y_2-y_1}{x_2-x_1}$ o.e. **ISW**	3 out of 4 substitutions correct Allow –0.75 $\frac{3}{-4}$ etc.
(iii)	Gradient of given line = $\frac{4}{3}$ $-\frac{3}{4}\times\frac{4}{3}=-1$ So lines are perpendicular	M1 B1ft B1	3 [7]	Attempt to rearrange equation to make y the subject **OR** attempt to find the gradient using points on the line Correct conclusion for their gradients States $-\frac{3}{4}\times\frac{4}{3}=-1$ or "negative reciprocal" relating to the correct values **www**	Must at least isolate y
2	$2x^3+9x^2-2px^2-9px+10x-10p$ $=2x^3+qx^2-8x-4q$ $p=2$ and $q=5$	M1* DM1 A1	3 [3]	Attempt to expand both sides **OR** to substitute 2 values of x into both expressions **OR** to express at least one side as a product of three factors Valid method to obtain either p or q Both values correct	If expanding, minimum of 5 terms on LHS and 3terms on RHS If comparing coefficients, must be of corresponding terms **SR** Spotted solutions **B1** one correct **B2** other correct
3 (i)	$8^{\frac{1}{2}}$	B1	1		Allow $8^{0.5}$ Condone p= $\frac{1}{2}$, just "$\frac{1}{2}$" seen as answer **www**
(ii)	8^{-2}	B1	1		Condone p = -2, just "-2" seen as answer **www** $\frac{1}{8^2}$ only not enough
(iii)	$2^8=\left(8^{\frac{1}{3}}\right)^8$ $=8^{\frac{8}{3}}$	M1 M1 A1	3 [5]	2^8 or $2^6=8^2$ soi $2=8^{\frac{1}{3}}$ soi o.e.	Condone p= $\frac{8}{3}$, just "$\frac{8}{3}$" seen as answer **www** 2^3= 8 not enough for second **M** mark

4	$u^2 - 5u + 4 = 0$	M1*		Use the given substitution to obtain a quadratic or factorise into 2 brackets each containing $(3x-2)^2$	**No marks** if evidence of "square rooting" e.g. "$(3x-2)^2$ - 5$(3x-2)$ + 2 (or 4) = 0" **No marks** if straight to quadratic formula to get x = "1" x = "4" and no further working
	$(u-1)(u-4) = 0$	DM1		Correct method to solve a quadratic	
	$u = 1$ or $u = 4$	A1		Correct values for u	**SR 1)** If **M0** Spotted solutions **www B1** each Justifies 4 solutions exactly **B2**
	$3x - 2 = \pm 1$ or $3x - 2 = \pm 2$	M1		Attempt to square root and rearrange to obtain x **OR** to expand, rearrange and solve quadratic (at least one)	**SR 2)** If first 3 marks awarded, spotted solutions 2 correct **B1** Other 2 correct **B1**
	$x = 1$ or $\frac{1}{3}$ or $\frac{4}{3}$ or 0	A1		2 correct values	Justifies 4 solutions exactly **B1**
		A1	6 [6]	All 4 correct values ($\frac{0}{3}$ = **A0**)	<u>Alternative scheme for candidates who multiply out:</u> Attempt to expand $(3x-2)^4$ and $(3x-2)^2$ **M1** $81x^4 - 216x^3 + 171x^2 - 36x = 0$ **A1** $x = 0$ a solution or x a factor of the quartic **A1** Attempt to use factor theorem to factorise their cubic **M1*** Correct method to solve quadratic **DM1** All 4 solutions correct **A1**
5 (i)		M1		Negative cubic through (0, 0) (may have max and min)	Must be continuous. Allow slight curve towards or away from y-axis at one end, but not both.
		A1	2	Must have reasonable rotational symmetry. Cannot be a finite "plot". Allow negative gradient at origin. Correct curvature at both ends.	
(ii)	$y = -(x-3)^3$	M1		$\pm(x-3)^3$ seen	
		A1	2	or $y = (3-x)^3$	Must have "$y =$ " for A mark **SR** $y = -(x-3)^2$ **B1**
(iii)	Stretch	B1			
	scale factor 5 parallel to y-axis	B1	2 [6]	o.e. e.g. scale factor $\frac{1}{\sqrt[3]{5}}$ parallel to the x axis.	Allow "factor" for "scale factor" For "parallel to the y axis" allow "vertically", "in the y direction". **Do not accept** "in/on/across/up/along the y axis"

6 (i)	$y = 5x^{-2} - \frac{1}{4}x^{-1} + x$	M1		x^{-2} used for $\frac{1}{x^2}$ **OR** x^{-1} used for $\frac{1}{x}$ soi, **OR** x correctly differentiated	Look out for: $y = 5x^{-2} - 4x^{-1} + x$ followed by $\frac{dy}{dx} = -10x^{-3} + 4x^{-2} + 1$ and then the correct answer.
	$\frac{dy}{dx} = -10x^{-3} + \frac{1}{4}x^{-2} + 1$	A1		kx^{-3} or kx^{-2} from differentiating	This is **M1 A1 A1 A0**
		A1		Two fully correct terms	$4x^{-1}$ is **NOT** a misread
		A1	**4**	Completely correct	
(ii)	$\frac{d^2y}{dx^2} = 30x^{-4} - \frac{1}{2}x^{-3}$	M1		Attempt to differentiate their $\frac{dy}{dx}$ (one term correctly differentiated)	Allow a sign slip in coefficient for M mark
		A1	**2** **6**	Completely correct	**NB** Only penalise "+ c" first time seen in the question

7 (i)	$4(x^2+3x)-3$ $=4\left[\left(x+\frac{3}{2}\right)^2-\frac{9}{4}\right]-3$ $=4\left(x+\frac{3}{2}\right)^2-12$	B1 B1 M1 A1	4	$p=4$ $q=\frac{3}{2}$ $r=-3-4q^2$ or $r=-\frac{3}{4}-q^2$ $r=-12$ (from $q=\pm1.5$)	If p, q, r found correctly, then **ISW** slips in format. $4(x+1.5)^2+12$ **B1 B1 M0 A0** $4(x+1.5)-12$ **B1 B1 M1 A1 (BOD)** $4(x+1.5x)^2-12$ **B1 B0 M1 A0** $4(x^2+1.5)^2-12$ **B1 B0 M1 A0** $4(x-1.5)^2-12$ **B1 B0 M1 A1** $4x(x+1.5)^2-12$ **B0 B1M1A1**
(ii)	$\frac{-12\pm\sqrt{12^2-4\times4\times-3}}{2\times4}$ $=\frac{-12\pm\sqrt{192}}{8}$ $=\frac{-12\pm8\sqrt{3}}{8}$ $=-\frac{3}{2}\pm\sqrt{3}$ OR: $4\left(x+\frac{3}{2}\right)^2-12=0$ $x+\frac{3}{2}=\pm\sqrt{3}$ $x=-\frac{3}{2}\pm\sqrt{3}$	M1 A1 B1 A1 M1 A1ft A1 A1	4	Correct method to solve quadratic $\frac{-12\pm\sqrt{192}}{8}$ or $\frac{-3\pm\sqrt{12}}{2}$ $\sqrt{192}=8\sqrt{3}$ or $\sqrt{12}=2\sqrt{3}$ from correct b^2-4ac $\frac{-3\pm2\sqrt{3}}{2}$ or $-\frac{12}{8}\pm\sqrt{3}$, $-\frac{6}{4}\pm\sqrt{3}$ Must have ± for method mark $x+1.5$ **ft** $x+q$ from part(i) www in LHS in part (ii) $\pm\sqrt{3}$ Do not **ISW**	Not for $2(x+q)=$ ... **SR** One correct root **www B1**
(iii)	$12^2-4\times4\times(-k)=0$ $144+16k=0$ $k=-9$ **OR (see next page)**	M1 A1 A1		Attempts $b^2-4ac=0$ or $\sqrt{b^2-4ac}=0$ involving k. If b^2-4ac not quoted then expression must be correct. Correct, unsimplified expression	<u>Other alternative methods</u> a) Attempt to factorise into two equal brackets, (may divide by 4 first – must be correct) **M1** Equate coefficient of x to 12 (or 3) **A1** $k=-9$ **A1** b) Uses differentiation to find x ordinate of turning point and uses this to form equation in k **M1** Correct equation in k **A1** $k=-9$ **A1**

7(iii) cont.	$4x^2+12x=k$ $4(x+\frac{3}{2})^2-9=k$	M1		Attempts completing the square in given equation or factorises to $(2x+3)^2-9=k$	Must involve k in their working to gain the method marks in this scheme
	Equal roots when $x=-\frac{3}{2}$	M1		Substitutes $x=-\frac{3}{2}$	
	$k=-9$	A1	3 [11]		
8 (i)	$\frac{dy}{dx}=6-2x$	M1 A1		Attempt to differentiate $\pm y$ Correct expression **cao**	One correct non-zero term
	When $x=5$, $6-2x=-4$	M1		Substitute $x=5$ into their $\frac{dy}{dx}$	
	When $x=5$, $y=12$	B1		Correct y coordinate	
	$y-12=-4(x-5)$	M1		Correct equation of straight line through (5, their y), their non-zero, numerical gradient	Allow $\frac{y-12}{x-5}$ = their gradient If using $y=mx+c$ must attempt at evaluating c
	$4x+y-32=0$	A1	6	Shows rearrangement to correct form	Allow any correct form e.g. $0=2y+8x-64$ etc.
(ii)	Q is point (8, 0)	B1ft		ft from line in (i)	.
	Midpoint of $PQ=\left(\frac{5+8}{2},\frac{12+0}{2}\right)$	M1		Uses $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ o.e. for their P,Q	
	$=\left(\frac{13}{2},6\right)$	A1	3		Do not accept $\left(\frac{13}{2},\frac{12}{2}\right)$
(iii)	$6-2x=0$	M1		Solution of their $\frac{dy}{dx}=0$	Alternatives for Method Mark a) attempts completion of square with $\pm(x-3)^2$ b) attempts to solve quadratic (usual scheme) and to find the mid-point of the two roots c) attempts to use $x=-\frac{b}{2a}$ (allow one sign slip on substitution)
	(Line of symmetry is) $x=3$	A1	2	Allow from $\pm[16-(x-3)^2]$, $\pm[6-2x=0]$	
(iv)	$x<3$	M1		$x<$ their3 or $x>$ their3 **OR** attempt to solve their $\frac{dy}{dx}>0$	May solve $\frac{dy}{dx}=0$ then use $\frac{d^2y}{dx^2}<0$ implies maximum point for the method mark, or sketch of curve Allow $x\le 3$
		A1	2 [13]	Allow from $\pm[16-(x-3)^2]$, $\pm[6-2x=0]$ in (iii)	

9 (i)	Centre (4, 1) $(x-4)^2+(y-1)^2-16-1-3=0$ $(x-4)^2+(y-1)^2=20$ Radius $=\sqrt{20}$	**B1** **M1** **A1**	 **3**	Correct centre Correct method to find r^2 Correct radius	 r^2 =(± their 4)^2 + (±their 1)^2 + 3 soi $\pm\sqrt{20}$ is **A0** Ignore incorrect simplification of $\sqrt{20}$
(ii)	$k=1\pm\sqrt{20}$ $k=1\pm 2\sqrt{5}$	**M1** **A1ft** **A1**	 **3**	y ordinate of their centre ± their radius or Both correct, unsimplified values **cao**	<u>Alternatives for method mark :</u> a) Substitutes k for y and uses $b^2-4ac=0$ to obtain quadratic in k b) Recognises $x = 4$ is equation of normal, substitutes into circle equation and solves for k. **SR** k = 1+ √20 or k = 1 – √20 or better **www B1**
(iii)	$MT^2=r^2-2^2$ $MT = 4$ $ST = 8$	**M1** **A1ft** **A1**	 **3**	Correct use of Pythagoras' theorem involving MT (or SM) Correct value of MT for their r **cao**	**SR** ST=8 from particular S and T co-ordinates [e.g. horizontal chord calculated as (0,3) and (8,3)] **B1** Justifies solution the same for all possible chords **B2**
(iv)	$x=2y+12$ $(2y+8)^2+(y-1)^2=20$ $4y^2+32y+64+y^2-2y+1=20$ $5y^2+30y+45=0$ $y^2+6y+9=0$ $(y+3)^2=0$ $y=-3$ $x=6$ **OR** $y-1=-2(x-4)$ Solve simultaneously with $y=\frac{1}{2}x-6$ $x=6$ $y=-3$ States line is tangent as meets at one point or verifies (6, -3) lies on circle	**M1*** **A1** **A1** **DM1** **A1** **A1** **M1** **A1** **M1** **A1** **A1** **B1**	 **6** **15**	Attempt to solve equations simultaneously Correct unsimplified expression, may be $(12+2y)^2+y^2-8(12+2y)-2y-3=0$ Obtain correct 3 term quadratic Correct method to solve quadratic of form $ax^2+bx+c=0$ $(b\neq 0)$ y value correct, no extra solutions x value correct **ISW** Attempt to find equation of radius/normal Correct equation Allow showing distance between (6,-3) and (4,1) = √20	Must be a clear attempt to reduce to one variable using equation of line and either form of equation of circle. Condone poor algebra for first mark. <u>If y eliminated:</u> $(x-4)^2+\left(\frac{1}{2}x-7\right)^2=20$ Or $x^2+\left(\frac{1}{2}x-6\right)^2-8x-2\left(\frac{1}{2}x-6\right)-3=0$ Leading to $x^2-12x+36=0$ **SR** Correct coordinates spotted or from trial and improvement **www B2**